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# Topological Aspect of Abelian Projected SU(2) Lattice Gauge Theory

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## Abstract

We show that the hypothesis of abelian dominance allows QCD-monopoles to preserve the topological feature of the QCD vacuum within SU(2) lattice gauge theory. An analytical study is made to find the relationship between the topological charge and QCD-monopoles in the lattice formulation. The topological charge is explicitly represented in terms of the monopole current and the abelian component of gauge fields in the abelian dominated system. We numerically examine the relation and demonstrate the abelian dominance in the topological structure by using Monte Carlo simulation.

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## I. INTRODUCTION

It is important to understand topological aspects of QCD to interpret several basic properties of the vacuum structure. Two distinct pictures of the QCD vacuum, which are based on the presence of topological objects; instanton and QCD-monopole, are now widely accepted. The main reason is that they give nice descriptions of several non-perturbative features, *e.g.* chiral symmetry breaking and color confinement. As is well known,  $SU(N_c)$  gauge theory has classical and non-trivial gauge configurations (instantons) satisfying the field equation in the 4-dimensional Euclidean space  $\mathbf{R}^4$  [1]. The topological charge, which corresponds to the non-trivial homotopy group  $\pi_3(SU(N_c)) = Z_\infty$ , is assigned to instanton configurations [1]. It has been established that such a topological feature plays an essential role on the resolution of the  $U_A(1)$  problem [2]. Furthermore, the instanton liquid characterized by a random ensemble of instanton and anti-instanton configurations succeeds in explaining several properties of light hadrons, *e.g.* spontaneous chiral-symmetry breaking (S $\chi$ SB) [1].

Another picture of the QCD vacuum is motivated from the stimulating idea of abelian gauge fixing, which was proposed by 't Hooft [3] and also independently by Ezawa-Iwazaki [4]. After performing a partial gauge fixing, which leaves an abelian gauge degree of freedom, one knows that point-like singularities in the three-dimensional space  $\mathbf{R}^3$ , associated with the homotopy group  $\pi_2(SU(N_c)/U(1)^{N_c-1}) = Z_\infty^{N_c-1}$ , can be identified as magnetic monopoles (QCD-monopoles) [3,4]. If monopoles are condensed in the QCD vacuum, the dual Meissner effect results [5]. Recent lattice QCD simulations actually show that the dual Meissner effect is brought to the true vacuum by QCD-monopole condensation<sup>†</sup> (see, *e.g.* a recent review article [6]). Hence, color confinement can be regarded as the dual version of

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<sup>†</sup>QCD-monopole condensation is characterized by the presence of the long and tangled monopole trajectories in the four-dimensional space  $\mathbf{R}^4$  and can be interpreted as the Kosterlitz-Thouless type phase transition.

superconductivity.

It seems that instantons and QCD-monopoles are relevant topological objects for the description of distinct phenomena. Here, we should emphasize that QCD-monopoles would play an essential role on non-perturbative features of QCD, which include not only confinement and but also  $S\chi$ SB. This possibility was first studied by using the Schwinger-Dyson equation with the gluon propagator in the background of condensed monopoles [7]. The idea of providing  $S\chi$ SB due to QCD-monopole condensation was supported by lattice simulations [8,9]. Thus, these results shed new light on the non-trivial relation between instantons and QCD-monopoles. Recently, both analytic works [10,11] and numerical works [12,13,15,14,16] have shown the existence of the strong correlation between these topological objects in spite of the fact that they originate from different homotopy groups. Furthermore, monopole trajectories become more complicated with the instanton density increasing in the background of a random ensemble of instanton solutions [17]. These results seem to suggest that the instanton liquid and QCD-monopole condensation may be indistinguishable.

In this paper, we would like to know whether the correlation between instantons and QCD-monopoles has some physical significance, or not, from the topological viewpoint. Here, we must not forget the hypothesis of abelian dominance, which Ezawa and Iwazaki had first advocated [4]. This hypothesis is essentially composed of two sentences [4]:

- Only the abelian component of gauge fields is relevant at a long-distance scale.
- The non-abelian effects are mostly inherited by magnetic monopoles.

Actually, lattice Monte Carlo simulation indicates that abelian dominance for some physical quantities, *e.g.* the string tension [18,19], the chiral condensate [8,9] and also several light hadron spectra [20,21] is realized in the maximally abelian (MA) gauge as well as monopole dominance. In this gauge, at least, the abelian component of the gauge field could be an important dynamical degree of freedom at a long-distance scale. Here, an unavoidable question arises relating to the non-trivial correlation between instantons and QCD-monopoles. *In the abelian dominated system, is it possible for the non-abelian topological nature to survive?* For

such an essential question, Ezawa and Iwazaki have proposed a remarkable conjecture [22]: once abelian dominance is postulated, the topological feature is preserved by the presence of monopoles. The main purpose of this paper is to confirm the presence of the non-trivial correlation between instantons and QCD-monopoles by finding evidence for the Ezawa-Iwazaki conjecture.

For simplicity, we restrict the discussion to the case of SU(2) gauge group throughout this paper. The organization of our paper is as follows. In Sec.2, we show that the topological charge is explicitly represented in terms of the monopole current and the abelian component of gauge fields through the hypothesis of abelian dominance. In Sec.3, we confirm numerically its justification by means of Monte Carlo simulation within SU(2) gauge theory after the MA gauge fixing. Finally, Sec.4 is devoted to a summary and conclusions.

## II. ABELIAN DOMINANCE HYPOTHESIS FOR TOPOLOGICAL CHARGE

We first address the definition of the topological charge in the lattice formulation. The naive and field-theoretical definition of the topological density [23] is given by

$$q(s) \equiv \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} \text{tr} \{ P_{\mu\nu}(s) P_{\rho\sigma}(s) \} , \quad (1)$$

with the clover averaged SU(2) plaquette:

$$P_{\mu\nu}(s) \equiv \frac{1}{4} ( U_{\mu\nu}(s) + U_{-\mu\nu}^\dagger(s) + U_{\mu-\nu}^\dagger(s) + U_{-\mu-\nu}(s) ) . \quad (2)$$

Here we have used the convenient notation for the SU(2) link variable;  $U_{-\mu}(s) = U_\mu^\dagger(s - \hat{\mu})$  so that  $U_{\pm\mu\pm\nu}(s) = U_{\pm\mu}(s) U_{\pm\nu}(s + \hat{\mu}) U_{\pm\mu}^\dagger(s + \hat{\nu}) U_{\pm\nu}^\dagger(s)$ . One naively expects that the topological charge  $Q_{\text{cont}}$  is extracted from the summation of the previously defined topological density over all sites up to leading order in powers of the lattice spacing  $a$  [23]:

$$Q_L = -\frac{1}{16\pi^2} \sum_s q(s) \simeq Q_{\text{cont}} + \mathcal{O}(a^6) , \quad (3)$$

where  $Q_{\text{cont}} = \frac{1}{16\pi^2} \sum_s \text{tr} \{ a^4 g^2 G_{\mu\nu}(s)^* G_{\mu\nu}(s) \}$ . Strictly speaking, the value  $Q_L$  has not only  $\mathcal{O}(a^6)$  corrections, but also a renormalized multiplicative correction of  $Q_{\text{cont}}$  in Eq.(3) [24]. In

general, one needs some smoothing method to eliminate undesirable ultraviolet fluctuations in the numerical simulation so that a renormalization factor would be brought to unity.

If we assume that the QCD vacuum is described as the abelian dominated system *in a suitable abelian gauge*, then the SU(2) link variable is expected to be U(1)-like as  $U_\mu(s) \simeq u_\mu(s) \equiv \exp\{i\sigma_3\theta_\mu(s)\}$ . Here, we define the angular variable  $\theta_\mu$  as

$$\theta_\mu(s) \equiv \arctan[U_\mu^3(s)/U_\mu^0(s)] \quad (4)$$

in the compact domain  $[-\pi, \pi]$  when the SU(2) link variable is parameterized as  $U_\mu(s) = U_\mu^0(s) + i\sigma_a U_\mu^a(s)$  ( $a = 1, 2, 3$ ) [25]. In the abelian dominated system, we might consider the abelian analog of the topological density  $q_{\text{Abel}}(s)$  [16], instead of the previously defined topological density, through the replacement of  $P_{\mu\nu}$  by the clover averaged U(1) plaquette  $p_{\mu\nu}$ :

$$\begin{aligned} p_{\mu\nu}(s) &\equiv \frac{1}{4} ( u_{\mu\nu}(s) + u_{-\mu\nu}^\dagger(s) + u_{\mu-\nu}^\dagger(s) + u_{-\mu-\nu}(s) ) \\ &= \frac{1}{4} \sum_{i,j=0}^1 u_{\mu\nu}(s - i\hat{\mu} - j\hat{\nu}) , \end{aligned} \quad (5)$$

where  $u_{\mu\nu}$  denotes the U(1) elementary plaquette. The explicit expression of  $q_{\text{Abel}}(s)$  is then given by

$$\begin{aligned} q_{\text{Abel}}(s) &= \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} \text{tr} \{ p_{\mu\nu}(s) p_{\rho\sigma}(s) \} \\ &= -\frac{1}{16} \sum_{i,j,k,l=0}^1 \varepsilon_{\mu\nu\rho\sigma} \sin\theta_{\mu\nu}(s - i\hat{\mu} - j\hat{\nu}) \sin\theta_{\rho\sigma}(s - k\hat{\rho} - l\hat{\sigma}) , \end{aligned} \quad (6)$$

where  $\theta_{\mu\nu}(s) \equiv \partial_\mu\theta_\nu(s) - \partial_\nu\theta_\mu(s)$  [26].  $\partial_\mu$  denotes the nearest-neighbor forward difference operator satisfying  $\partial_\mu f(s) \equiv f(s + \hat{\mu}) - f(s)$ .

Our next aim is to discuss the expression of the abelian analog of the topological density in the naive continuum limit  $a \rightarrow 0$ . This is because we need only the leading order term in powers of the lattice spacing to determine the corresponding topological charge. Here, one may notice that the U(1) elementary plaquette  $u_{\mu\nu}$  is a multiple valued function of the U(1) plaquette angle  $\theta_{\mu\nu}$ . Hence, we divide  $\theta_{\mu\nu}$  into two parts,

$$\theta_{\mu\nu}(s) = \bar{\theta}_{\mu\nu}(s) + 2\pi n_{\mu\nu}(s) , \quad (7)$$

where  $\bar{\theta}_{\mu\nu}$  is defined in the principal domain  $[-\pi, \pi]$ , which corresponds to the U(1) field strength. The anti-symmetric tensor  $n_{\mu\nu}$  can take the restricted integer values  $0, \pm 1, \pm 2$ . Taking the limit  $a \rightarrow 0$ , *i.e.*  $\bar{\theta}_{\mu\nu} \rightarrow 0$ , we thus arrive at the following expression [26]:

$$q_{\text{Abel}}(s) \approx -\varepsilon_{\mu\nu\rho\sigma} \bar{\Theta}_{\mu\nu}(s) \bar{\Theta}_{\rho\sigma}(s) , \quad (8)$$

where  $\bar{\Theta}_{\mu\nu}(s) \equiv \frac{1}{4} \sum_{i,j=0}^1 \bar{\theta}_{\mu\nu}(s - i\hat{\mu} - j\hat{\nu})$ . The r.h.s of Eq.(8) does not necessarily vanish in spite of the fact that it is represented only in terms of the U(1) field strength.

Next, we show that the non-vanishing contribution to the value of  $q_{\text{Abel}}(s)$  results from monopoles. For the identification of monopoles, we follow DeGrand-Toussaint's definition in the compact U(1) gauge theory [27]. The magnetic current is given by

$$\begin{aligned} k_\mu(s) &\equiv \frac{1}{4\pi} \epsilon_{\mu\nu\rho\sigma} \partial_\nu \bar{\theta}_{\rho\sigma}(s + \hat{\mu}) \\ &= -\frac{1}{2} \epsilon_{\mu\nu\rho\sigma} \partial_\nu n_{\rho\sigma}(n + \hat{\mu}) . \end{aligned} \quad (9)$$

In the last line, we have used the Bianchi identity on the U(1) plaquette angle;  $\epsilon_{\mu\nu\rho\sigma} \partial_\nu \theta_{\rho\sigma} = 0$ . Then we can easily see that the current  $k_\mu$  denotes the integer-valued magnetic current, *i.e.* the monopole current [27]. Strictly speaking, the monopole current resides at the dual lattice site orthogonal to direction  $\mu$ . For simplicity, we have used an undiscriminating notation between the ordinary lattice site and the dual lattice site. One may notice that it is obvious that  $k_\mu$  is topologically conserved;  $\partial'_\mu k_\mu = 0$ .  $\partial'$  denotes the nearest-neighbor backward difference operator satisfying  $\partial'_\mu f(s) \equiv f(s) - f(s - \hat{\mu})$ .

Here, we define the averaged magnetic current<sup>†</sup> in the  $2^3$  extended cube at the center of the lattice site  $s$  orthogonal to direction  $\mu$  [26] as

$$\mathcal{K}_\mu(s) \equiv \frac{1}{8} \sum_{i,j,k=0}^1 k_\mu(s - i\hat{\nu} - j\hat{\rho} - k\hat{\sigma}) , \quad (10)$$

<sup>†</sup>In some sense, this is similar to the type-II extended monopole current [28].

where there indices  $(\nu, \rho, \sigma)$  are complementary to  $\mu$ . Using the nearest-neighbor central difference operator;  $\Delta_\mu \equiv \frac{1}{2}\{\partial_\mu + \partial'_\mu\}$ , we therefore rewrite  $\mathcal{K}_\mu$  as the following form:

$$\mathcal{K}_\mu(s) = \frac{1}{4\pi} \epsilon_{\mu\nu\rho\sigma} \Delta_\nu \bar{\Theta}_{\rho\sigma}(s) , \quad (11)$$

which obviously satisfies the conservation law;  $\Delta_\mu \mathcal{K}_\mu(s) = 0$ .

To show the explicit contribution of monopoles to the topological charge, we introduce the dual potential  $\mathcal{B}_\mu$  satisfying the following equation [29]:

$$(\Delta^2 \delta_{\mu\nu} - \Delta_\mu \Delta_\nu) \mathcal{B}_\nu(s) = -2\pi \mathcal{K}_\mu(s) . \quad (12)$$

where  $\Delta^2 \equiv \Delta_\mu \Delta_\mu$ . We can perform the Hodge decomposition on the clover-average U(1) field strength  $\bar{\Theta}_{\mu\nu}$  with the dual potential  $\mathcal{B}_\mu$  as

$$\bar{\Theta}_{\mu\nu}(s) = \Delta_\mu \mathcal{A}_\nu(s) - \Delta_\nu \mathcal{A}_\mu(s) + \epsilon_{\mu\nu\rho\sigma} \Delta_\rho \mathcal{B}_\sigma(s) , \quad (13)$$

where  $\mathcal{A}_\mu$  is the Gaussian fluctuation [29]. After a little algebra using the partial summation, we find the explicit contribution of monopoles to the r.h.s of Eq.(8) [26] as

$$\begin{aligned} \epsilon_{\mu\nu\rho\sigma} \bar{\Theta}_{\mu\nu}(s) \bar{\Theta}_{\rho\sigma}(s) &= 4\epsilon_{\mu\nu\lambda\omega} \epsilon_{\lambda\omega\rho\sigma} \Delta_\mu \mathcal{A}_\nu(s) \Delta_\rho \mathcal{B}_\sigma(s) \\ &= 16\pi \mathcal{A}_\mu(s) \mathcal{K}_\mu(s) + \dots , \end{aligned} \quad (14)$$

where the ellipsis stands for the total divergence, which will drop in the summation over all sites. Consequently, we arrive at the conjecture that *the topological feature is preserved by the presence of monopoles in the abelian dominated system* [26]:

$$Q_{\text{cont}} \simeq -\frac{1}{16\pi^2} \sum_s q_{\text{Mono}}(s) , \quad (15)$$

where  $q_{\text{Mono}}(s) \equiv -16\pi \mathcal{A}_\mu(s) \mathcal{K}_\mu(s)$ . In addition, the Gaussian fluctuation can not be definitely identified except for the Landau gauge solution;  $\Delta_\mu \mathcal{A}_\mu^L(s) = 0$  where a superscript  $L$  denotes ‘Landau’. However, we need only to know the Gaussian fluctuation in the Landau gauge on the measurement of the quantity  $\sum_s q_{\text{Mono}}(s)$ , which has the residual U(1) gauge invariance.

### III. NUMERICAL RESULTS

This section is devoted to a numerical analysis to justify the above conjecture through Monte Carlo simulation within SU(2) lattice gauge theory using the standard Wilson action. We choose the MA gauge as an applicable abelian gauge for the realization of abelian dominance in this paper. As we pointed out, the QCD vacuum has been observed as an abelian dominated system by lattice Monte Carlo simulations [6]. We shall return to the explicit procedure of the MA gauge fixing later.

As we have mentioned before, one needs to smooth the Monte Carlo gauge configurations in order to determine the topological charge. To eliminate undesirable fluctuations, we adopt the naive cooling method which is an iterative scheme to replace each link variable using the following procedure [23]:

$$U_\mu(s) \rightarrow U'_\mu(s) = \Sigma_\mu(s)/\|\Sigma_\mu(s)\| , \quad (16)$$

where  $\|\Sigma_\mu(s)\|$  denotes the square root of determinant of  $\Sigma_\mu$  and

$$\Sigma_\mu(s) = \sum_{\nu \neq \mu} \left( U_\nu(s) U_\mu(s + \hat{\nu}) U_\nu^\dagger(s + \hat{\mu}) + U_\nu^\dagger(s - \hat{\nu}) U_\mu(s - \hat{\nu}) U_\nu(s + \hat{\mu} - \hat{\nu}) \right) . \quad (17)$$

This procedure is derived from the condition to minimize the action through the variation of the link variable [23]. Then, an iterated process leads to a local minimum of the action, which corresponds to a solution to the field equation. The relation  $Q_L \approx Q_{\text{cont}}$  is expected through the cooling procedure.

Before turning to the abelian gauge fixing, we must take account of the permutation between the cooling procedure and the gauge fixing procedure. The action in the compact lattice gauge theory is not usually affected by the gauge fixing procedure for the link variables because of the gauge-invariant measure. However, the fundamental modular region [30] appears in the case of the MA gauge fixing [31]. Then, the action is modified by the contribution from the Faddeev-Popov determinant [31]. In other words, the above cooling procedure is not justified after the MA gauge fixing. Thus, we have to apply the MA gauge fixing procedure to the resulting gauge configuration after several cooling sweeps.

In the lattice formulation, the MA gauge fixing is defined by maximizing the gauge dependent variable  $R$  under gauge transformations;  $U_\mu(s) \rightarrow \tilde{U}_\mu(s) = \Omega(s)U_\mu(s)\Omega^\dagger(s + \hat{\mu})$  [25]:

$$R[\Omega] = \sum_{s, \mu} \text{tr} \left\{ \sigma_3 \tilde{U}_\mu(s) \sigma_3 \tilde{U}_\mu^\dagger(s) \right\} = 2 \sum_{s, \mu} (1 - 2\chi_\mu(s)) , \quad (18)$$

where  $\chi_\mu \equiv (\tilde{U}_\mu^1(s))^2 + (\tilde{U}_\mu^2(s))^2$ . The maximization of  $R$  implies that the off-diagonal components of the SU(2) link variables are minimized as much as possible through the gauge transformation.

After the MA gauge fixing, we extract the U(1) field strength  $\bar{\theta}_{\mu\nu}$  and the monopole current  $k_\mu$  from the U(1) link variable following the previous section. The Gaussian fluctuation in the Landau gauge is computed by the convolution of the electric current with the lattice Coulomb propagator  $\mathcal{D}(s - s')$

$$\mathcal{A}_\mu^L(s) = - \sum_{s'} \mathcal{D}(s - s') \Delta_\lambda \bar{\Theta}_{\lambda\mu}(s') , \quad (19)$$

where the lattice Coulomb propagator satisfies the equation;  $\Delta^2 \mathcal{D}(s - s') = -\delta_{s, s'}$ . One can obtain the explicit form of  $\mathcal{D}(s - s')$  on a  $L^4$  lattice as

$$\mathcal{D}(s - s') = \sum_p \frac{1}{\sum_\mu \sin^2(p_\mu)} e^{ip \cdot (s - s')} \quad (20)$$

with the following abbreviation:  $p_\mu \equiv \frac{2\pi}{L} n_\mu$  and  $\sum_p \equiv \frac{1}{L^4} \prod_\mu \sum_{n_\mu=0}^{L-1}$ .

We generate the gauge configurations by using the heat bath algorithm on a  $16^4$  lattice at  $\beta = 2.4$ . All measurements have been performed on independent configurations, which are separated from each other by 500 sweeps after the system has reached thermal equilibrium for 1000 heat bath sweeps. For the MA gauge fixing in the numerical simulation, we use the overrelaxation method with parameter  $\omega = 1.7$  [32]. Let us concentrate on the realization of the abelian dominance for topological density in the MA gauge. Before turning to the topological density, we show the probability distribution of the amplitude of the off-diagonal component  $\chi_\mu$  in Fig.1. The solid curve, the broken curve and the dotted curve denote the case for configurations before cooling and after cooling for 1 sweep and 30 sweeps,

respectively. This figure tells us that the off diagonal components of the SU(2) link variable are actually made as small as possible on the whole lattice in the MA gauge. Furthermore, such a tendency becomes prominent with more cooling sweeps. The information from Fig.1 suggests that large numbers of cooling sweeps could enhance abelian dominance. In addition, the off diagonal components;  $U_\mu^1$  and  $U_\mu^2$  behave like random variables on the whole lattice before the MA gauge fixing so that the probability distribution is flat on unity in the case of no gauge fixing whether configurations are cooled, or not.

We have checked the above expectation on the measurement of the topological density  $q(s)$  and the abelian analog of topological density  $q_{\text{Abel}}(s)$ . Fig.2 and Fig.3 display  $q(s)$  and  $q_{\text{Abel}}(s)$  in a two-dimensional plane after 30 and 100 cooling sweeps. We define this plane as a slice at the center of the instanton in the  $(z, t)$ -plane. The center is identified as the local maximum of  $q(s)$ . The topological density around the instanton is little affected by the cooling procedure as is shown in Fig.2(a) and Fig.2(b). For the abelian analog of topological density, such a stability in the cooling process is not required because of the lack of the topological basis in the U(1) manifold. At 30 cooling sweeps, the abelian dominance for topological density is not largely revealed as shown in Fig.3(a), since a large amplitude of the off-diagonal components still remains locally, especially around the center of the instanton and/or the monopole. However, after 100 cooling sweeps, one can recognize that a similar lump to the topological density is located around the center of the instanton in Fig.3(b). Thus, the topological density is dominated by the abelian analog of topological density after enough cooling sweeps, as expected.

Next, we will examine the correlation between topological charge and monopoles. The two types of corresponding topological charge are computed through the following formula:

$$Q_{\text{SU}(2)} \equiv -\frac{1}{16\pi^2} \sum_s q(s) , \quad (21)$$

$$Q_{\text{Mono}} \equiv -\frac{1}{16\pi^2} \sum_s q_{\text{Mono}}(s) . \quad (22)$$

$Q_{\text{SU}(2)}$  denotes the ordinary topological charge.  $Q_{\text{Mono}}$  denotes the corresponding topological charge via monopoles. We show the scatter plots of  $Q_{\text{SU}(2)}$  vs.  $Q_{\text{Mono}}$  in Fig.4; (a) no

gauge fixing and (b) the MA gauge fixing after 100 cooling sweeps for 50 independent configurations. Obviously, there is not any correlation between the two topological charges *before the MA gauge fixing*. A one-to-one correspondence between  $Q_{\text{SU}(2)}$  and  $Q_{\text{Mono}}$  is revealed in the scatter plot *after the MA gauge fixing*. In further detail, we find that there is a small variance between  $Q_{\text{SU}(2)}$  and  $Q_{\text{Mono}}$ . The slope of the linear correlation in the scatter plot is not unity, but 1.43 in Fig.4(b) ( $16^4$  at  $\beta = 2.4$ ). On the several measurements at  $\beta = 2.45$  and 2.5, it seems that this slope hardly depends on the lattice spacing after large enough numbers of cooling sweeps (see Table 1). Table 1 tells us that the relation;  $Q_{\text{Mono}} \approx 0.7Q_{\text{SU}(2)}$  is almost satisfied in the MA gauge on these data.

$\beta$	cooling sweeps		
	30	50	100
2.40	1.45	1.46	1.43
2.45	1.46	1.43	1.44
2.50	1.49	1.43	1.43

To discuss the topological feature on  $Q_{\text{Mono}}$ , we show the probability distribution of  $Q_{\text{Mono}}$  in Fig.5 by using 3000 independent configurations at  $\beta = 2.4$  on a  $16^4$  lattice. Several dotted lines correspond to partial contributions to the whole distribution, which are assigned to some integer value of  $Q_{\text{SU}(2)}$ . We find several discrete peaks in Fig.5, since each partial contribution is a Gaussian-type distribution with the half width slightly less than unity around discrete values ( $\approx 0.7Q_{\text{SU}(2)}$ ). This implies that  $Q_{\text{Mono}}$  is classified by approximately discrete values. Namely, it seems that monopoles almost inherit the topological nature of the original gauge configurations.

#### IV. SUMMARY AND CONCLUSION

Our work was motivated by the Ezawa-Iwazaki conjecture: the topological feature is preserved by the presence of QCD-monopoles once abelian dominance is postulated. To confirm this, we have performed an analytical study of how the monopole current is related to the topological charge within SU(2) lattice gauge theory. We consider the abelian analog of topological density, which is defined through the replacement of the SU(2) link variable by the U(1)-like link variable. Although this quantity is represented by the corresponding U(1) field strength in the naive continuum limit, it can carry the non-trivial contribution to the topological charge because of the presence of QCD-monopoles. If we assume that the abelian component of gauge fields plays an essential role in the description of the long range physics, QCD-monopoles are required to inherit the topological feature.

Monte Carlo simulations have brought us encouraging results for the above conjecture. We have chosen the MA gauge as an applicable abelian gauge. We have measured both the ordinary topological charge  $Q_{\text{SU}(2)}$  and the corresponding topological charge via monopoles,  $Q_{\text{Mono}}$ . In spite of the fact that there is no correlation between  $Q_{\text{SU}(2)}$  and  $Q_{\text{Mono}}$  without abelian gauge fixing, we found a one-to-one correspondence between the two charges in the MA gauge. And also the relation:  $Q_{\text{Mono}} \approx 0.7Q_{\text{SU}(2)}$  can be read off from our resulting data. Furthermore,  $Q_{\text{Mono}}$  is classified by approximately discrete values. Thus we have concluded that the topological nature is substantially inherited by QCD-monopoles. This conclusion is consistent with our previous study [33], which shows that QCD-monopoles strongly correlate with the presence of fermionic zero-modes in the MA gauge.

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## FIGURE CAPTIONS

**FIG.1.** The probability distribution of the amplitude of the off-diagonal components  $\chi$  on a  $16^4$  lattice at  $\beta = 2.4$ .

**FIG.2.** The topological density as a function of  $z$  and  $t$  in a two dimensional slice at the center of the instanton after 30 cooling sweeps (a), and after 100 cooling sweeps (b) at  $\beta = 2.4$  on a  $16^4$  lattice.

**FIG.3.** The abelian analog of topological density as a function of  $z$  and  $t$  in a two dimensional slice at the center of the instanton after 30 cooling sweeps (a), and after 100 cooling sweeps (b) at  $\beta = 2.4$  on a  $16^4$  lattice.

**FIG.4.** Scatter plot of  $Q_{\text{SU}(2)}$  vs.  $Q_{\text{Mono}}$  (a) before the MA gauge fixing and (b) after the MA gauge fixing at 100 cooling sweeps.

**FIG.5.** Probability distribution of  $Q_{\text{Mono}}$  in the MA gauge fixing at 100 cooling sweeps by using 3000 independent configurations.

**TABLE 1.** Slope of the linear correlation in the scatter plot of  $Q_{\text{SU}(2)}$  vs.  $Q_{\text{Mono}}$  on a  $16^4$  lattice at  $\beta = 2.4, 2.45$  and  $2.5$  after 30, 50 and 100 cooling sweeps.

FIG.1 (Phys.Rev.D) Shoichi Sasaki *et al.*

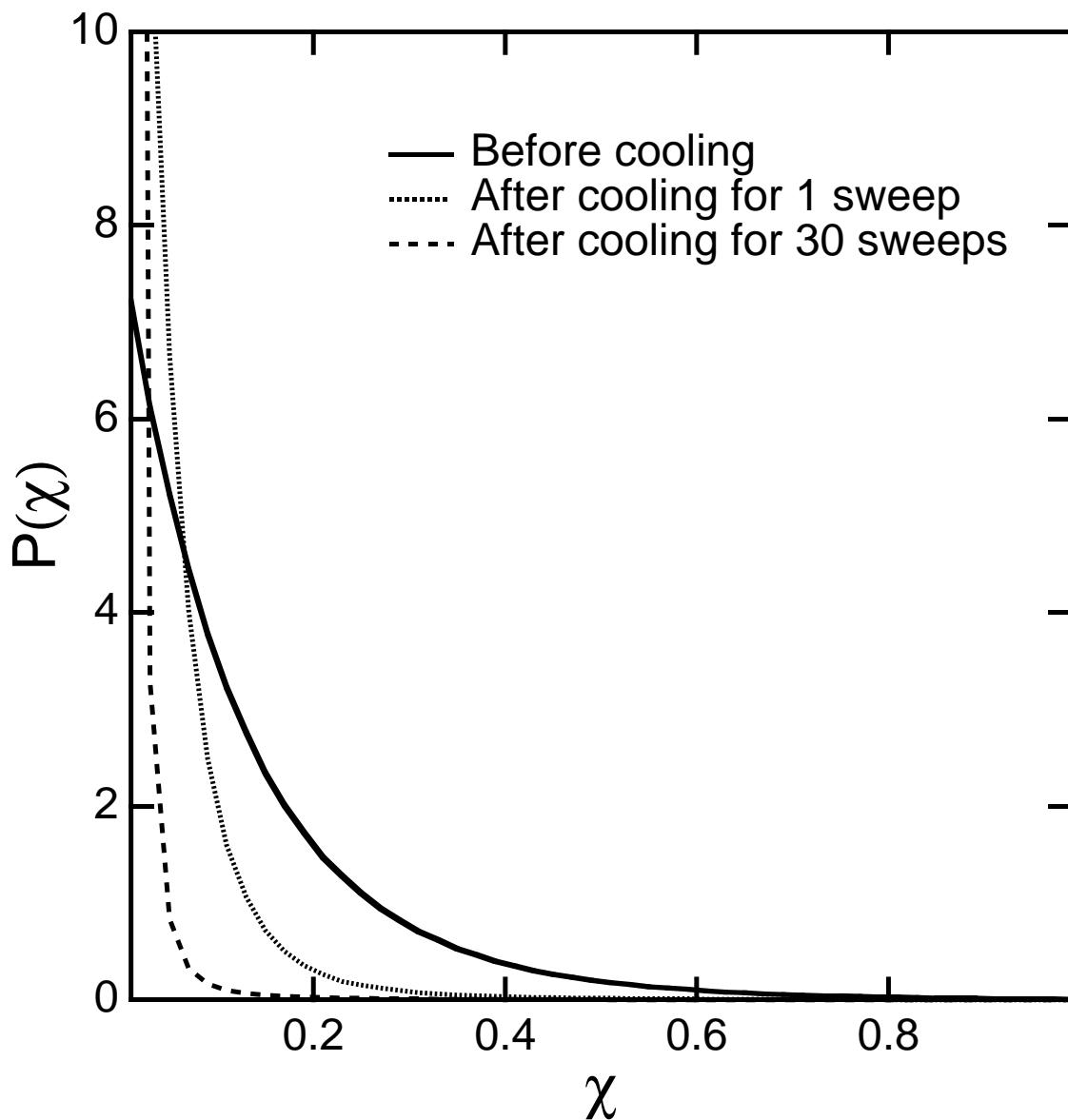


Fig. 1

FIG.2a (Phys.Rev.D) Shoichi Sasaki *et al.*

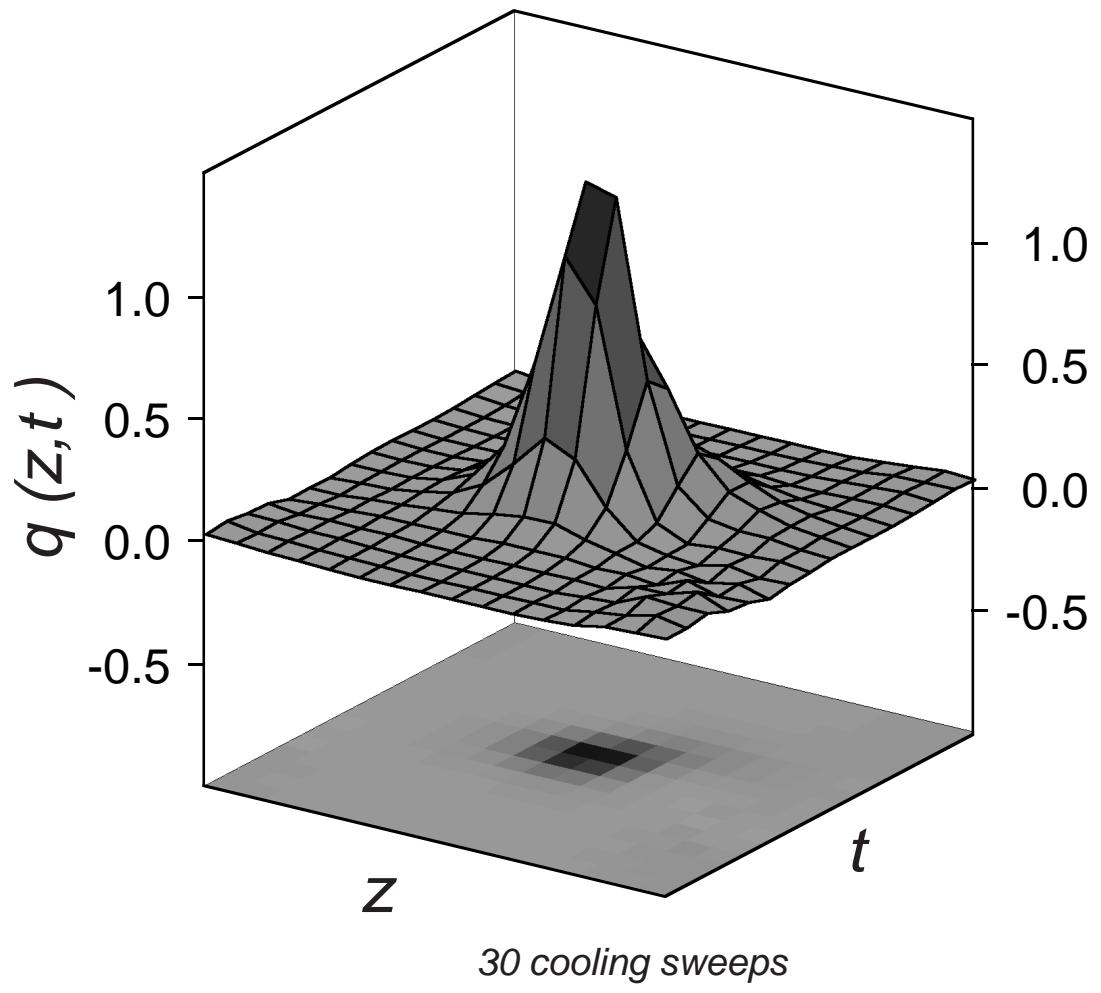


Fig. 2(a)

FIG.2b (Phys.Rev.D) Shoichi Sasaki *et al.*

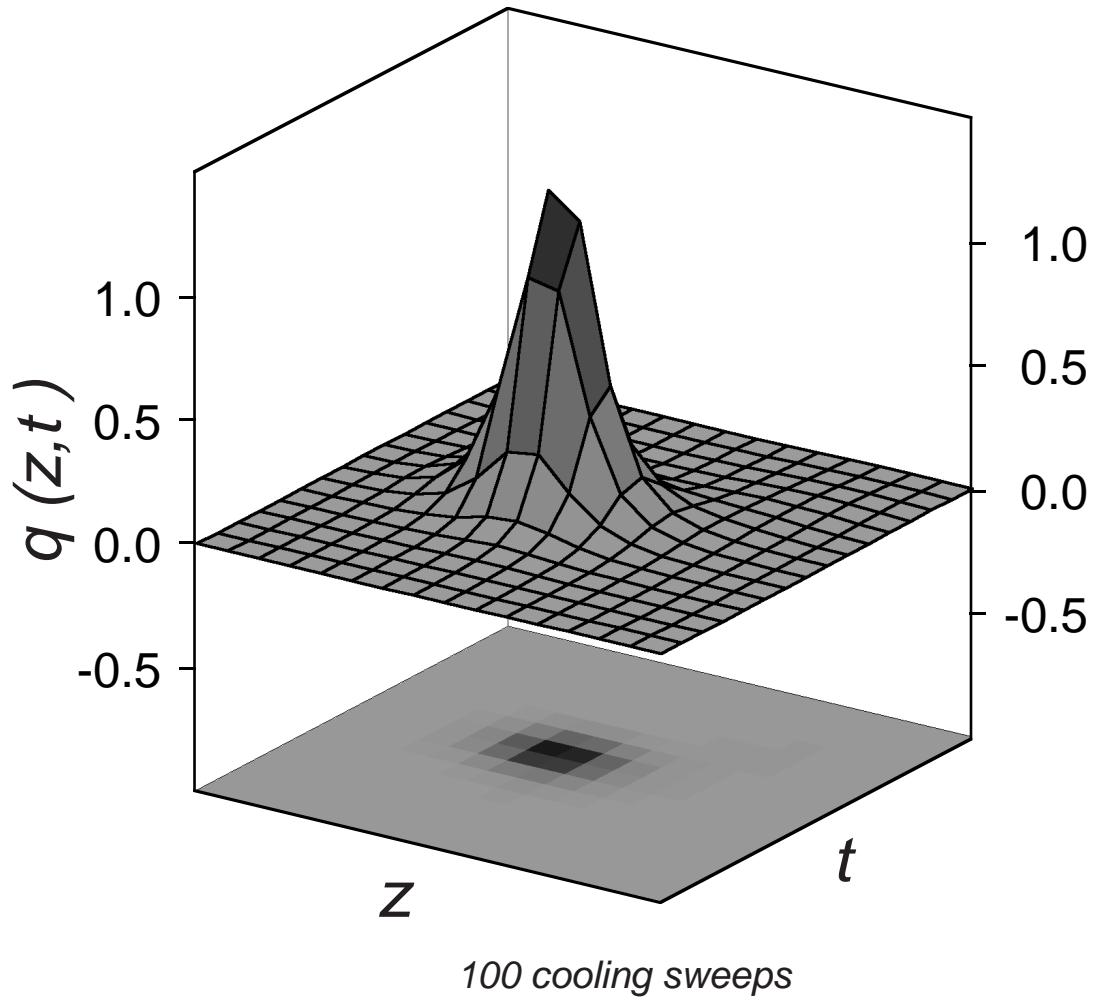


Fig. 2(b)

FIG.3a (Phys.Rev.D) Shoichi Sasaki *et al.*

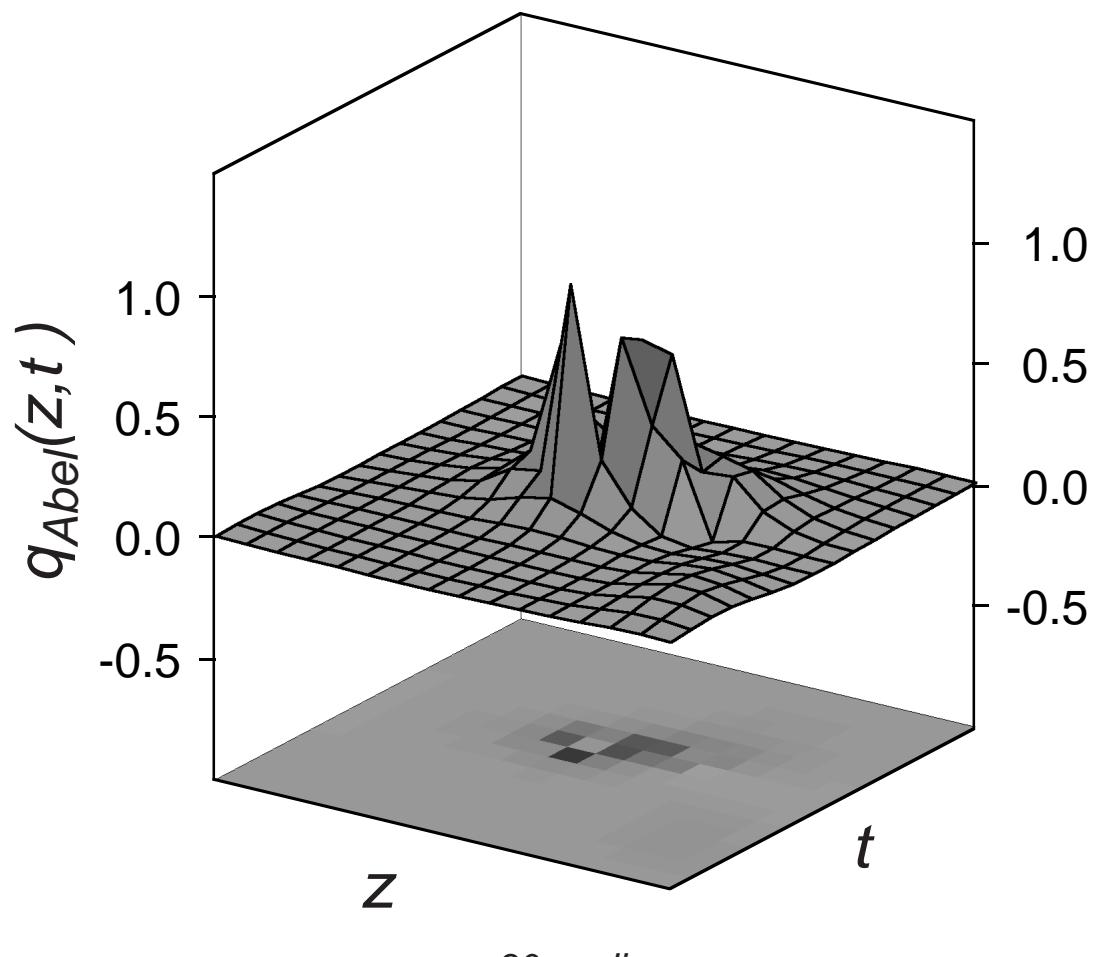


Fig. 3(a)

FIG.3b (Phys.Rev.D) Shoichi Sasaki *et al.*

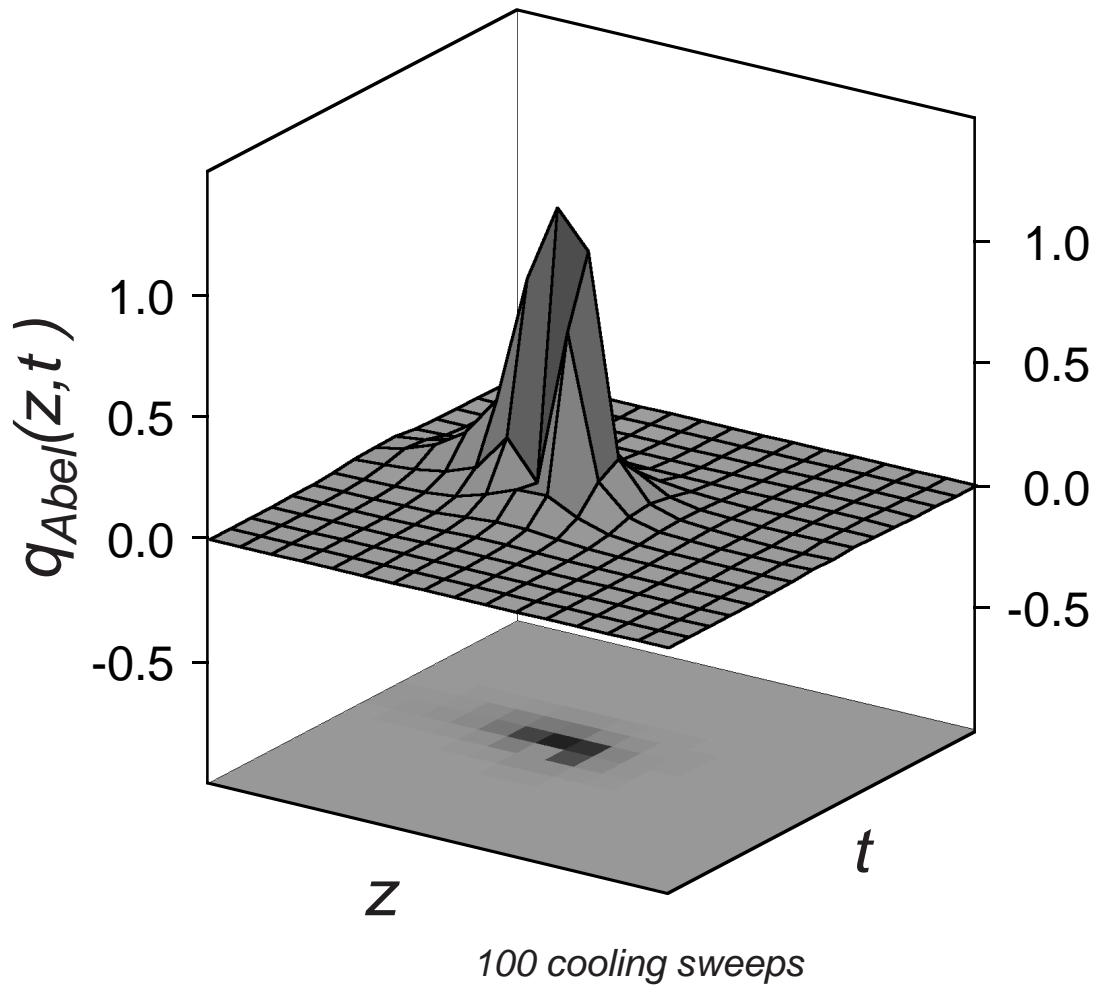


Fig. 3(b)

FIG.4a (Phys.Rev.D) Shoichi Sasaki *et al.*

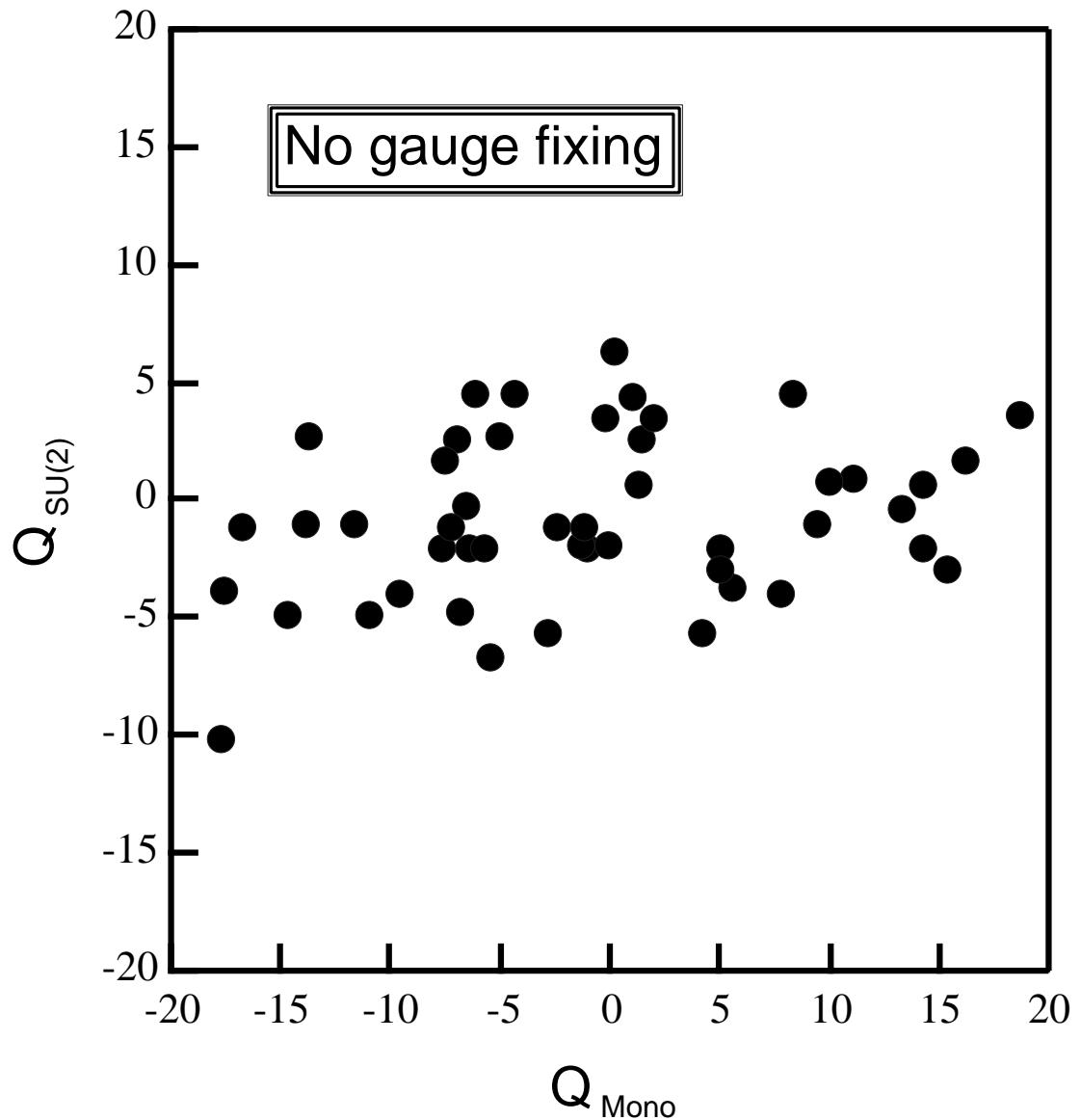


Fig. 4(a)

FIG.4b (Phys.Rev.D) Shoichi Sasaki *et al.*

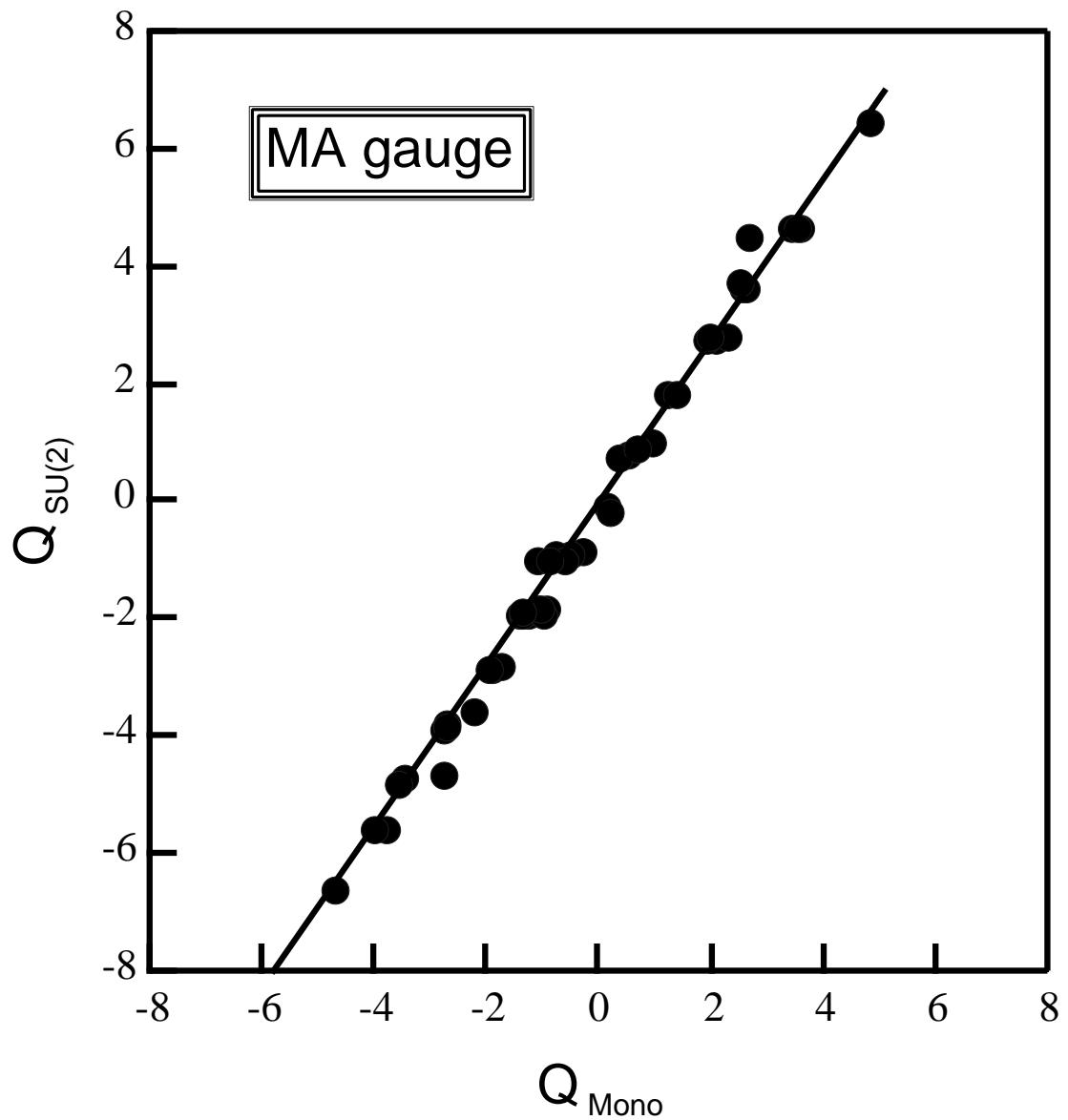


Fig. 4(b)

FIG.5 (Phys.Rev.D) Shoichi Sasaki *et al.*

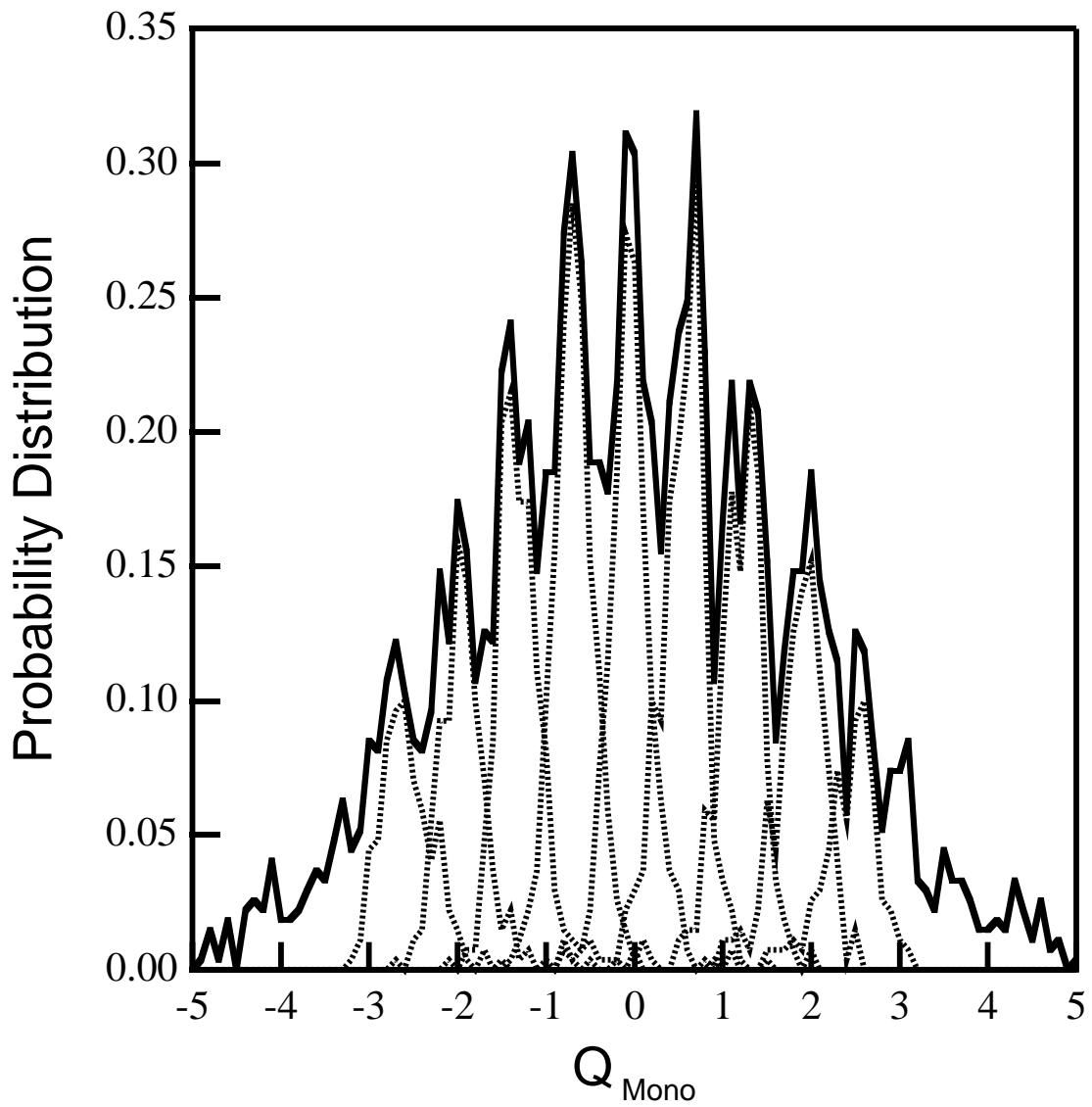


Fig. 5

## REFERENCES

- [1] For a recent review article, see T. Schäfer and E.V. Shuryak, Rev. Mod. Phys. **70** (1998) 323.
- [2] G. 't Hooft, Phys. Rep. **142** (1986) 357.
- [3] G. 't Hooft, Nucl. Phys. **B190** (1981) 455.
- [4] Z.F. Ezawa and A. Iwazaki, Phys. Rev. **D25** (1982) 2681.
- [5] G. 't Hooft, in High energy physics, ed. A. Zichichi (Editions Compositori, Bologna, 1975);  
S. Mandelstam, Phys. Rep. **23** (1976) 245.
- [6] M.I. Polikarpov, Nucl. Phys. **B** (Proc. Suppl.) **53** (1997) 134, and references therein.
- [7] S. Sasaki, H. Suganuma and H. Toki, Prog. Theor. Phys. **94** (1995) 373.  
H. Suganuma, S. Sasaki and H. Toki, Nucl. Phys. **B435** (1995) 207.
- [8] O. Miyamura, Phys. Lett. **B353** (1995) 91.
- [9] R.M. Woloshyn, Phys. Rev. **D51** (1995) 6411;  
F.X. Lee, R.M. Woloshyn, H.D. Trottier, Phys. Rev. **D53** (1996) 1532.
- [10] R.C. Brower, K.N. Orginos and C.-I. Tan, Phys. Rev. **D55** (1997) 6313.
- [11] H. Suganuma, A. Tanaka, S. Sasaki and O. Miyamura, Nucl. Phys. **B** (Proc. Suppl.) **47** (1996) 302;  
H. Suganuma, S. Umisedo, S. Sasaki, H. Toki and O. Miyamura, Aust. J. Phys. **50** (1997) 233.
- [12] O. Miyamura and S. Origuchi, *QCD-monopoles and Chiral Symmetry Breaking on SU(2) Lattice*, in: Proc. Int. RCNP Workshop on Color Confinement and Hadrons 'Confinement 95', eds. H. Toki, Y. Mizuno, H. Suganuma, T. Suzuki and O. Miyamura (World

Scientific, 1995), p.235.

[13] S. Thurner, M. Feurstein, H. Markum and W. Sakuler, Phys. Rev. **D54** (1996) 3457;  
 S. Thurner, M.C. Feurstein and H. Markum, Phys. Rev. **D56** (1997) 4039;  
 M. Feurstein, H. Markum and S. Thurner, Phys. Lett. **B396** (1997) 203.

[14] H. Suganuma, S. Sasaki, H. Ichie, F. Araki and O. Miyamura, Nucl. Phys. **B**  
 (Proc. Suppl.) **53** (1997) 528.

[15] A. Hart and M. Teper, Phys. Lett. **B371** (1996) 261.

[16] V. Bornyakov and G. Schierholz, Phys. Lett. **B384** (1996) 190.

[17] M. Fukushima, S. Sasaki, H. Suganuma, A. Tanaka, H. Toki and D. Diakonov, Phys.  
 Lett. **B399** (1997) 141.

[18] T. Suzuki and I. Yotsuyanagi, Phys. Rev. **D42** (1990) 4257.

[19] G.S. Bali, V. Bornyakov, M. Müller-Preussker and K. Schilling, Phys. Rev. **D54** (1996)  
 2863;  
 J.D. Stack, S.D. Neiman and R.J. Wensley, Phys. Rev. **D50** (1994) 3399.

[20] O. Miyamura and S. Origuchi, Nucl. Phys. **B** (Proc. Suppl.) **47** (1996) 314.

[21] S. Kitahara, O. Miyamura, T. Okude, F. Shoji and T. Suzuki, Nucl. Phys. **B533** (1998)  
 576.

[22] Z.F. Ezawa and A. Iwazaki, Phys. Rev. **D26** (1982) 631.

[23] E.-M. Ilgenfritz, M.L. Laursen, M. Müller-Preußker, G. Schierholz and H. Schiller, Nucl.  
 Phys. **B268** (1986) 693.

[24] M. Campostrini, A. Di Giacomo and H. Panagopoulos, Phys. Lett. **B212** (1988) 206.

[25] A.S. Kronfeld, G. Schierholz and U.-J. Wiese, Nucl. Phys. **B293** (1987) 461;

A.S. Kronfeld, M.L. Laursen, G. Schierholz and U.-J. Wiese, Phys. Lett. **B198** (1987) 516.

[26] S. Sasaki and O. Miyamura, *A Conclusive Test of Abelian Dominance Hypothesis for Topological Charge in the QCD Vacuum*, preprint YITP-98-56, hep-lat.9808019, to be published in Proc of Lattice 98.

[27] T.A. DeGrand and D. Toussaint, Phys. Rev. **D22** (1980) 2478.

[28] T.L. Ivanenko, A.V. Pochinsky and M.I. Polikarpov, Phys. Lett. **B252** (1990) 631.

[29] J. Smit and A.J. van der Sijs, Nucl. Phys. **B355** (1991) 603.

[30] D. Zwanziger, Nucl. Phys. **B378** (1992) 525.

[31] M.N. Chernodub, M.I. Polikarpov and A.I. Veselov, Phys. Lett. **B342** (1995) 303.

[32] J.E. Mandula and M. Ogilvie, Phys. Lett. **B248** (1990) 156.

[33] S. Sasaki and O. Miyamura, Phys. Lett. **B443** (1998) 331;  
 S. Sasaki and O. Miyamura, Nucl. Phys. **B** (Proc. Suppl.) **63** (1998) 507.